

Equivalent fractions

How many fractions are there equivalent to : $\frac{4}{10}$

1) $\frac{1}{5}$

$$\frac{8}{20}$$

$$10$$

2) $\frac{2}{5}$

$$\frac{10}{25}$$

$$\frac{20}{50}$$

$$\frac{64}{160}$$

$$\frac{40}{100}$$

Cancelling Fractions

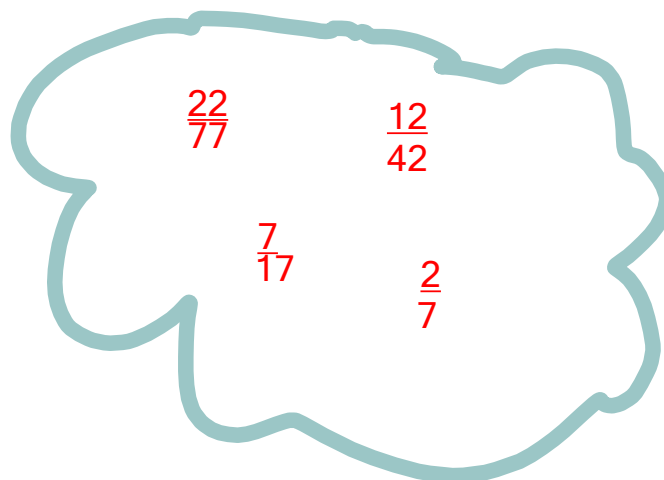
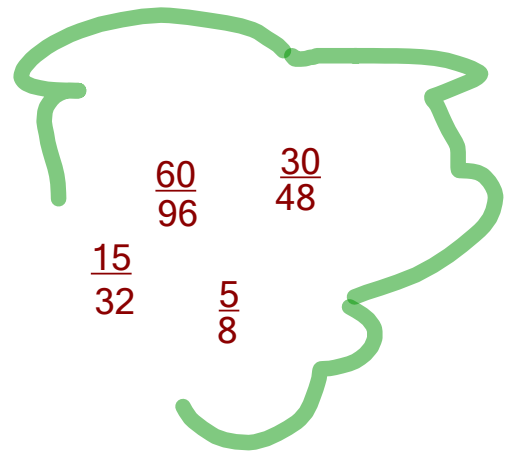
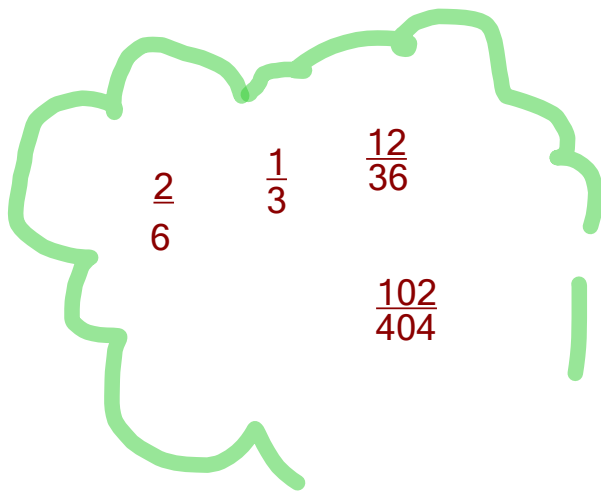
$$\frac{\cancel{4}}{\cancel{12}} = \frac{2}{6} = \frac{1}{3}$$

Cancel these fractions to their lowest terms

$$\frac{20}{45} = \frac{4}{9} \quad \frac{5}{45} = \frac{1}{9} \quad \frac{40}{90} = \frac{4}{9}$$

$$\frac{27}{42} = \frac{9}{14} \quad \frac{16}{96} = \frac{4}{24} = \frac{1}{6} \quad \frac{120}{600} = \frac{2}{10} = \frac{1}{5}$$

Which of these fractions is the odd man out?



Ordering fractions

There are two main ways to order fractions:

- if you can use a calculator, change them to decimals.
- use equivalent fractions to compare.

Put these fractions in order of size, starting with the smallest:

1) $\frac{5}{6}$ $\frac{2}{3}$ $\frac{3}{10}$ $\frac{1}{2}$ no calculator

2) $\frac{4}{33} = \frac{8}{66}$ $\frac{6}{11} = \frac{36}{66}$ $\frac{1}{2} = \frac{33}{66}$ $\frac{5}{6} = \frac{55}{66}$ no calculator

3) $\frac{1}{4}$, $\frac{2}{5}$, $\frac{1}{6}$, $\frac{2}{7}$ $\frac{1}{6}$, $\frac{1}{4}$, $\frac{2}{7}$, $\frac{2}{5}$

4) $\frac{5}{8}$, $\frac{3}{7}$, $\frac{4}{9}$, $\frac{3}{10}$

5) $\frac{7}{11}$, $\frac{4}{15}$, $\frac{6}{12}$, $\frac{5}{7}$

6) $\frac{1}{6}$, $\frac{2}{9}$, $\frac{5}{12}$, $\frac{7}{15}$

Finding a fraction of an amount.

$\frac{5}{8}$ of 88 means find one eighth, then find 5 of those

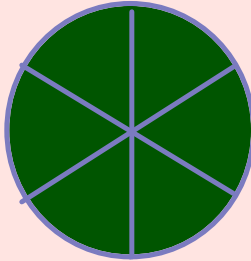
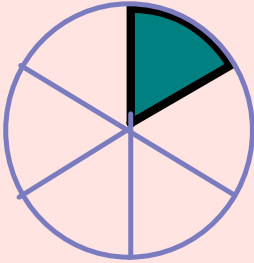
work out:

$$\frac{3}{7} \text{ of } 35 = 15$$

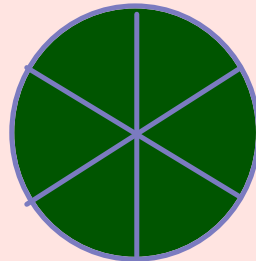
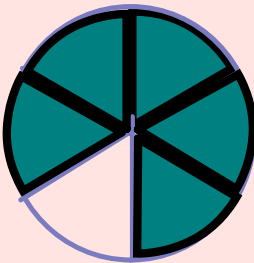
$$\frac{4}{5} \text{ of } 45 = 36$$

$$\frac{2}{9} \text{ of } 36 = 8$$

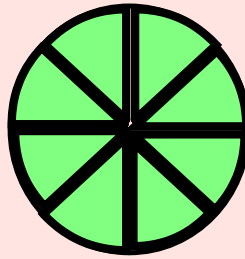
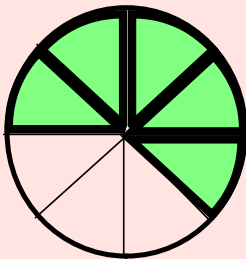
Improper Fractions



$$1\frac{1}{6} = \frac{7}{6}$$



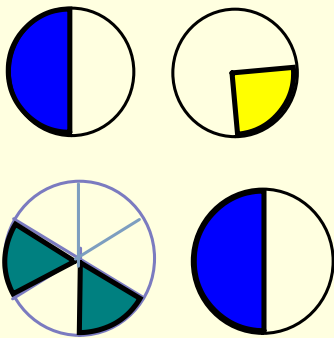
$$1\frac{5}{6} = \frac{11}{6}$$



Draw a diagram to show $1\frac{2}{3}$

Draw a diagram to show $\frac{7}{5}$

Adding Fractions with different denominators



$$\frac{1}{4} + \frac{1}{8}$$

1. $\frac{2}{3} + \frac{1}{6} =$

2. $\frac{3}{4} + \frac{1}{8} =$

3. $\frac{3}{10} + \frac{2}{5} =$

4. $\frac{2}{5} + \frac{3}{15} =$

5. $\frac{3}{4} + \frac{1}{12} =$

6. $\frac{1}{2} + \frac{1}{3} =$

$\frac{2}{5} + \frac{3}{7} =$

How to add and subtract fractions without going to pieces

1. Find a common denominator, the lower the better.
2. Using your common denominator make equivalent fractions.
3. Add (or subtract) the numerators.
4. Simplify.

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Multiplying Fractions

**This is the easiest of all the fraction questions.
Multiply the numerators together.
Multiply the denominators together.
That's it!**

$$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$$

If you have a whole number, change it to an improper fraction:

$$2 \frac{1}{3} \times \frac{3}{4} =$$

A fraction trick: when multiplying fractions, cancel where you can and you will save yourself work.

For example : $\frac{2}{3} \times 9 =$

$$45 \times \frac{4}{9} =$$

$$60 \times \frac{2}{3} =$$

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Dividing by a fraction

**This is easy if you can remember the rule:
Turn the second fraction upside down and
multiply!**

Find 3 fractions that have a sum of 1, but all have different denominators.

$$\frac{1}{2} + \frac{1}{4} + \frac{2}{8}$$

$$\frac{2}{4} + \frac{2}{8} + \frac{4}{16}$$

$$\frac{1}{3} + \frac{2}{6} + \frac{3}{9}$$

$$\frac{1}{2} + \frac{1024}{4096} + \frac{2048}{8192}$$

$$\frac{3}{6} + \frac{1}{4} + \frac{2}{8}$$

$$\frac{1}{3} + \frac{1}{2} + \frac{1}{6}$$

$$1\frac{2}{3} \div \frac{1}{2} = \frac{5}{3} \times 2 = \frac{10}{3} = 3\frac{1}{3}$$

$$2\frac{3}{4} \div \frac{7}{9} = \frac{11}{4} \cdot \frac{9}{7} = \frac{11}{4} \times \frac{9}{7} = \frac{99}{28} \\ = 3\frac{15}{28}$$

$$3\frac{2}{5} \div 2\frac{1}{8} = \frac{17}{5} \div \frac{17}{8} \\ = \frac{17}{5} \times \frac{8}{17} = \frac{8}{5} = 1\frac{3}{5}$$

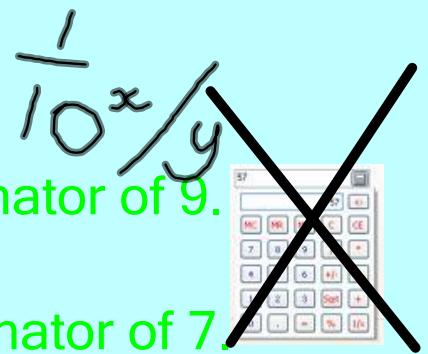
Converting fractions to decimals

Which fractions do you know the decimal equivalent of?

Terminating decimals	Recurring Decimals
$\frac{1}{2} = 0.5$	$\frac{1}{3} = 0.33333\dots$
$\frac{1}{4} = 0.25$	$\frac{1}{7} = 0.142857$
$\frac{1}{8} = 0.125$	$\frac{1}{9} = 0.1111$
$\frac{1}{10} = 0.1$	$\frac{1}{6} = 0.16666$
$\frac{1}{5} = 0.2$	$\frac{2}{3} = 0.6666$

Try to find a rule for fractions which terminate.

Test it out on the fractions:



Investigate fractions with a denominator of 9.

Investigate fractions with a denominator of 7.

Can this recurring decimal be expressed as a fraction? $0.678678678\dots$

$$\text{Let } x = 0.678678678$$

$$1000x = 678.678678$$

$$x = 0.678678$$

$$\begin{array}{r} 1000x = 678.678678 \\ x = 0.678678 \\ \hline 999x = 678 \end{array} \quad x = \frac{678}{999} = \frac{226}{333}$$

$$x = 0.4545454545\dots$$

$$100x = 45.4545$$

$$x = 0.4545$$

$$x = \frac{45}{99} = \frac{5}{11}$$

$$\hline 99x = 45$$

$$x = 3.153153153\dots$$

$$1000x = 3153.153153$$

$$x = 3.153153$$

$$\hline 999x = 3150$$

$$x = \frac{3150}{999}$$

$$= 3\frac{153}{999}$$

$$= 3\frac{51}{333}$$

$$= 3\frac{17}{111}$$

$$\text{Let } x = 7.405140514051\dots$$

$$10000x = 74051.4051\dots$$

$$x = 7.4051\dots$$

$$\hline 9999x = 74044$$

$$x = \frac{74044}{9999}$$

$$= 7\frac{4051}{9999}$$

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$$x = 0.5555\dots$$

$$\left. \begin{array}{l} 10x = 5.5555\dots \\ x = 0.5555\dots \end{array} \right\}$$

$$9x = 5$$

$$x = \frac{5}{9}$$

$$\begin{array}{r} x = 0.16666 \text{ ---} \\ 100x = 16.6666 \\ \hline x = 0.16666 \\ \hline 99x = 16.5 \end{array}$$

$$990x = 165$$

$$x = \frac{165}{990} = \frac{33}{198} = \frac{11}{66} = \frac{1}{6}$$

$$x = 0.23232323$$

$$100x = 23.2323\dots$$

$$x = 0.2323$$

$$99x = 23$$

$$x = \frac{23}{99}$$

Changing recurring decimals to fractions

$$0.36363636\dots$$

$$\text{let } x = 0.36363636\dots$$

$$10x = 3.63636$$

$$100x = 36.36363636 \quad \left. \begin{array}{l} 10x = 3.63636 \\ 100x = 36.36363636 \end{array} \right\} -$$

$$x = 0.36363636$$

$$99x = 36$$

$$x = \frac{36}{99}$$

$$x = \frac{12}{33} = \frac{4}{11}$$

$$x = 0.041\dot{6}$$

$$1000x = 41.6$$

$$\left. \begin{array}{l} \frac{1}{7} = 0.14285\dot{7} \\ x = 0.142857 \\ 1000000x = 142857.142857\dots \\ x = 0.142857\dots \end{array} \right\} -$$

$$999999x = 142857$$

$$x = \frac{142857}{999999}$$

$$= \frac{15873}{111111} = \frac{1}{7}$$

A1, A2, A4

A5 read a)+b) do c)

A6 B1 →

$$\frac{1}{3} = 0.\dot{3}$$

$$\frac{2}{3} = 0.\dot{6} +$$

$$\frac{3}{3} = 0.\dot{9} = 1$$

$$\frac{4}{5}x = \frac{3}{7} \qquad \frac{3}{8}x = \frac{33}{48}$$

$$\frac{5}{8}x = \frac{15}{40} \qquad \frac{7}{9}x = 1\frac{2}{3}$$

$$\frac{4}{5}x = \frac{3}{7}$$

$$1 = \frac{3}{7} \times \frac{5}{4}$$

$$x = \frac{15}{28}$$

$$\frac{3}{8}x = \frac{33}{48}$$

$$x = \frac{33}{48} \cdot \frac{8}{3}$$

$$x = \frac{33 \cdot 8}{48 \cdot 3}$$

$$x = \frac{11}{6}$$

$$\frac{5}{8}x = \frac{15}{40}$$

$$x = \frac{15}{40} \div \frac{5}{8}$$

$$= \frac{15}{40} \times \frac{8}{5}$$

$$= \frac{3}{5}$$

$$\frac{7}{9}x = 1\frac{2}{3}$$

$$\frac{7}{9}x = \frac{5}{3}$$

$$x = \frac{5}{3} \times \frac{9}{7}$$

$$x = \frac{15}{7} = 2\frac{1}{7}$$

Convert 0.316̄ to a fraction.

$$x = 0.31\dot{6}$$

$$100x = 31.6$$

$$1000x = 316.6$$

$$\underline{- 100x = 31.6}$$

$$900x = 285$$

$$x = \frac{285}{900} = \frac{57}{180} = \frac{19}{60}$$

$$31.\dot{6}$$

$$\underline{- 0.31\dot{6}}$$

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Homework: convert these decimals to fractions:

- 1) 0.56̄ 2) 0.605̄ 3) 0.45̄ 4) 3.7826̄

$\frac{5}{6}$ of a school day is spent in lessons.

What percentage of the day is spent in lessons?

72% of students like sport.

What fraction of the students like sport?

Solving problems with fractions.

1. A cake is divided up between 3 people. Alice has two fifths, Tim has two sevenths and Christian has the rest.

- How much does Christian get?
- Who has the biggest piece?
- If Christian gave half his piece to Tim how much would they each get?

2. A bag of sweets is shared between 4 people. Filipa has one fifth, Robyn has one fifteenth, Sarah has one third and Louisa has 60 sweets. How many sweets did they have?

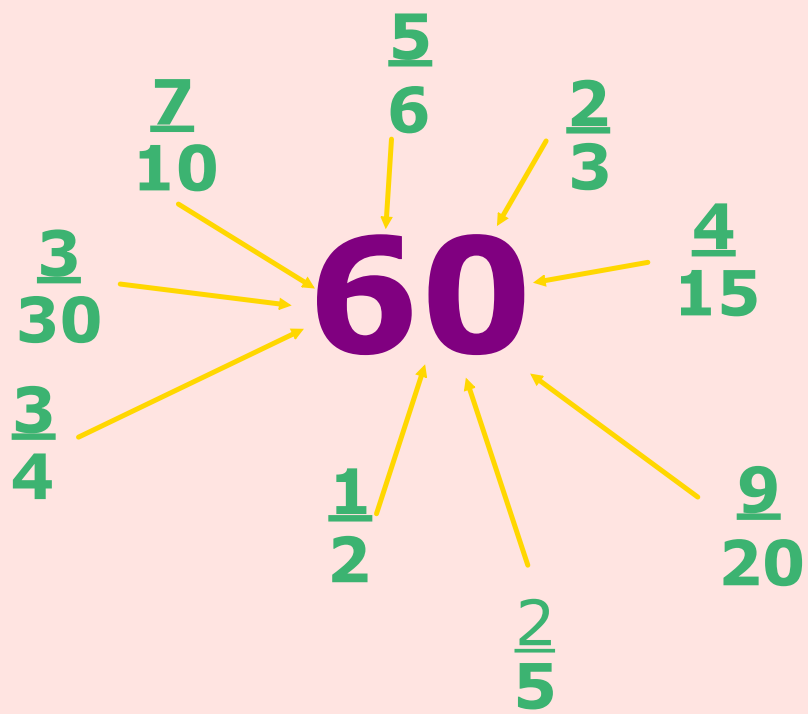
3. A school day of 6 hours is divided into modules of three-quarters of an hour. How many modules are there in a day?

4. (extension) The Egyptians used fractions but only those with a numerator of one (called unit fractions). They would not repeat a fraction in working.

How might they have made $\frac{5}{8}$?

What about $\frac{7}{12}$?

Work out:



understand equivalent fractions, simplifying a fraction by cancelling all common factors
order fractions by rewriting them with a common denominator
calculate a given fraction of a given quantity, expressing the answer as a fraction
express a given number as a fraction of another
add and subtract fractions by writing them with a common denominator
perform short division to convert a simple fraction to a decimal
addition, subtraction, multiplication and division of mixed numbers
multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction
distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals), and other fractions (which are represented by recurring decimals)
convert a recurring decimal to a fraction
multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction
convert simple fractions of a whole to percentages of the whole and vice versa
use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact answer